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A METHODOLOGY FOR DECISION ASSUMPTION MEASUREMENT

by

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I. Introduction

The purpose of this paper is to motivate and briefly summarize a methodology for explicit measurement of the assumptions which underly decisions.[1] This methodology was developed for use in managerial planning and control contexts, but is also suitable for use by operations researchers in developing descriptive models and for use in marketing research.

Central Problem Context

The evolution of managerial planning and control processes has included improvements across four components: (a) managerial models of problem situations, (b) the methodology of making individual decisions, (c) procedures of self-observation and learning, and (d) procedures for improved communication and coordination among the participants within an organization.

Such advances have been and are likely to be in the future importantly dependent on increased explicitness in planning and control.

One might imagine a scenario in which individual managers explicitly record their objectives, methodology, and assumptions associated with particularly important or repetitive decisions. In this scenario, the manager is familiar with procedures useful for eliciting from himself in a natural manner this kind of explicitness. He thus has a record of his decision basis which he compares in detail with his changing objectives and assumptions through time as the decision differ significantly from his expectations. The existence of this explicit "model" allows the manager to do an analysis of the total outcome variance into sub-variances; for example, he might discover which of his planning assumptions had been mistaken. Such pin-pointing allows him to generalize better the knowledge gained from the analysis to other decisions

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[1] Much of this paper is based on work described in the author's unpublished doctoral dissertation; see Wilcox [25].

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in planning future activities as well as for re-planning and controlling on-going activities resulting from past plans.

In this scenario the manager also communicates portions of this record to other members of the organization. Consequent benefits are more congruent inter-personal expectations, permitting larger organizational units and reduced internal friction. Some of the planning knowledge which he acquires is made available for the use of other managers with related problems or for training purposes. Thus, a considerable fraction of the planning knowledge gained during the course of the individual's experience accrues permanently to the organization.

The foregoing scenario is not reality and it is not necessarily a prediction. A particularly serious obstacle is that managers do not yet know how to be very explicit, even when they are trying.

This phenomena is familiar to those who ask managers or consumers why they choose particular decision alternatives. What one generally finds in many such cases is that the answers either are not put in operational terms or else the specified operations would not, in fact, lead to the decision patterns observed. This is very often true even when no intent to deceive is present.

The measurement methodology to be presented here makes some progress toward overcoming this obstacle by bringing relatively easily obtained data concerning decision assumptions to the manager's attention. This methodology provides the potential basis for attaining personalized information systems which may provide both the information most useful to the user given his existing assumptions and that most useful in helping him to revise his assumptions.

Other Problem Contexts

A second, though clearly related, problem context arises from the attempt of the operations researcher or information system specialist to model the manager's decision, either normatively or descriptively. Too often to be ignored, the resulting model is of little value because the operations researcher was unable to discover and successfully model a key decisional determinant.[2] Although narrowness of scope may sometimes be thought to be appropriate on the grounds that the model is normative and need not take into account "irrational" influences, this "normative" argument is many times merely a convenient rationalization. In addition, even when the normative model is valid, its implementation often founders because of its lack of relation to the existing assumptions of the manager in authority. What is needed here is a way in which the staff specialist can easily and objectively assimilate and accomodate the manager's assumptions.

A third problem context is that of measuring common decision assumptions of large numbers of consumer decision-makers, such as is done in market research. The need here is readily apparent. Methodologies like the one described here, but with certain relative disadvantages, are already beginning to come into use for this purpose.

II. Some Pre-existing Measurement Methodologies for Assumptions

One may describe a wide class of assumptions, to which we will restrict ourselves here, in terms of the following three elements:

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[2] See Little [14], Ackoff [1].

1. a first attribute position or comparison
2. a relation
3. a second attribute position or comparison.

Measuring such an assumption may be done more easily if two of these elements can be regarded as given. For example, suppose we wished to measure the assumptions of three different decision-makers regarding "favorable changes in" (the relation) "the likelihood of a future increase in the price of XYZ Corporation's common stock" (the second attribute position). Note that their assumptions as applied to the object, XYZ Corporation, must differ, if at all, in the implicit first attribute position or comparison. However, in this they may differ in two distinct ways:

1. in the position of XYZ Corporation along particular first attributes common to two or three decision-makers;
2. in the first attribute used to characterize XYZ Corporation.

For example, one decision-maker might assume that a recent increase in XYZ Corporation's marketing capability resulted in a favorable change in the likelihood of a future increase in the price of its common stock, while the second might assume that a recent decrease in XYZ's marketing capability resulted in an unfavorable change in such a likelihood. At one level, their assumptions are the same: that recent increases of a firm's marketing capability result in a more favorable likelihood of a future increase in the price of the firm's common stock, and vice versa. At a sub-level, they differ because they ascribe different positions of XYZ Corporation on the first attribute, "recent changes in marketing capability". Consider, in contrast, the

difference between these assumptions and one by a third decision-maker that "recent positive increases in the price of XYZ's stock resulted in a favorable change in the likelihood of a future increase in the price of that stock". Here the first attribute used to characterize XYZ's situation is along a dimension entirely different than that employed by the other two decision-makers. Thus, measuring the attribute dimension used to characterize an object as well as the object's position on the dimension is often crucial.

Measurement Procedures

How can one measure assumption dimension? One obvious method is direct questioning of the decision-maker; however, this approach is of limited usefulness because we often cannot rely on the decision-maker to give an objective and accurate assessment of his own, perhaps partly unconscious assumptions. This is a problem even when the decision-maker himself is asking the questions. When an outside observer asks the questions, his preconceptions limit reliability.

The alternative approach is through indirect means. We might seek, as did H. Simon and his associates, merely to observe the decision-makers' informational inputs and decision outputs over a period of time, perhaps combined with asking him to "think aloud". This is an expensive, extremely time consuming proposition requiring a high level of skill by the observer. Another reasonable approach is for an outside observer to pre-specify a large number of potentially relevant input attributes and use observation and regression analysis to narrow the list. However, for really individualized assumptions this will be inefficient at best and often miss the point entirely.[3] We

[3] See Clarkson [3] for a description of "think aloud" process and its results in modeling the assumptions of a single decision-maker engaged in investment portfolio choice. See Slovic [22] for the simple regression or analysis of variance approach on a similar problem.

might try standardized psychological testing methods. Unfortunately, these will tend to be limited to assumptions regarding a large number of events or objects. However, they are much less expensive and more objective. Two which offer promise here are the semantic differential and the various multi-dimensional scaling methods (MDS).[4]

In brief, the semantic differential is based on a factor analysis of a large number of decision-makers' ratings of objects on a number of more or less relevant adjective scales. These objects can then be automatically rated on the more fundamental attribute factors, if any, revealed by the factor analysis of the ratings by all the decision-makers on all the scales. A comparison of the factor scores of the objects with independent data regarding some other attribute, such as the decision-maker's preferences for the objects, could then give a measure of the decision-maker's assumptions relating the factor attributes to the independent attribute.

This use of the semantic differential, though more efficient than straightforward regression because it first reduces the attributes to factors before trying to relate them to preferences, faces several important problems. First, combining the decision-makers' ratings before factor-analysis obscures, rather than elicits, individual differences in attributes used. Second, there is the difficulty in making the adjective scales sufficiently specific and relevant to the problem at hand, and third, the method of rating objects on a standard seven or nine point scale makes rather strong assumptions about the metric nature of these adjective scales. If one performed a separate factor analysis for each decision-maker one could better get at differences between persons in

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[4] See Osgood, et al. [18], for a description of the semantic differential, Harmon [7], for a review of factor-analysis. See Silk [21] for a relatively recent discussion of multi-dimensional scaling.

the attribute structure used to characterize the same set of objects. However, this has not been done extensively, partly because meaningful, statistically significant factors are hard to obtain using a reasonably small number of objects to be rated on a large number of partly irrelevant adjective scales. Aggregating the responses of a number of people may eliminate the latter statistical problem but obliterates individual differences. One thing clearly needed is a way of selecting relevant adjective scales before the factor analysis.

The second method, that of multidimensional scaling (MDS) and especially its newer non-metric variants, on the other hand, does not face the problems of the semantic differential. MDS methods use estimates or comparisons of inter-object similarities to build up a spatial configuration of objects. The particular configuration determines the minimum number of dimensions of the space in which it can be "easily" embedded. Criteria similar in role to the rotational criteria for factor analysis determine the axes of the most useful coordinate system in this space. Again, the resulting coordinates of objects can be associated with preferences to measure assumptions. In contrast to the semantic differential, no pre-specification of attributes is necessary and only very weak ordinal assumptions as to the types of comparisons of similarity are required. These two important advantages make multi-dimensional scaling much better suited for investigation of individual differences in attribute dimensions used to characterize objects.[5]

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[5] See Shepard [20] and Kruska [13] for a description of the breakthrough that led to the new, powerful methods for multidimensional scaling which have been developed in the last decade. See Tucker and Messick [23] for early work on individual differences.

Studies using MDS have demonstrated important differences among decision-makers in the attribute dimensions they use to characterize a set of objects.[6] Such multi-dimensional scaling measures could be combined with measures on a known rating or preference scale to determine assumptions. In fact, this has already been done.[7]

For example, Cliff has related multi-dimensional scaling measures of the attribute structure used by students in describing fields of academic study to their ratings of likings for these fields. Rigney and De Bow similarly related dimensions used by military officers to characterize simulations of attacks to their ratings of the degree of threat implicit in such attacks. Paul Green et al. have used MDS to relate attributes used by business school students to characterize business firms and their preference ratings of the firms' common stocks. An interesting recent study illustrating this methodology to a management science audience is that by Klahr.

Though the technical feasibility of MDS for assumption measurement has thus been suggested, there appear to be remaining difficulties which the methodology to be described in the remainder of this paper appears to substantially overcome.

Despite their elegance, multidimensional scaling methods seem relatively infeasible for moderately complicated practical situations. First, they have required quite large numbers of interstimulus comparisons in order to construct a metric of adequate dimensions. This would not be a major difficulty in itself,

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[6] See Horan [9] and Warr, Schroder, and Blackman [24] for eye-opening recent studies regarding such individual differences.

[7] See Cliff [4], Rigney and De Bow [19], and Green and Maheshwari [6]. See also Klahr [12].

if it were not likely that the decision-maker's task of combining similarities on various more or less independent attributes into a summary similarity response for pairs or triads of stimulus objects imposes additional work of deciding on combinatorial weights.

A second serious difficulty, that of interpreting the resulting dimensions, is aggravated because MDS discards available introspective data on the labels of intermediate attributes used in forming the summary similarity judgements.

While this problem of interpretation of dimensions by an outside observer is not unique to MDS, its seriousness is usually much less in semantic differential or other factor analytic methods because we have available the loadings of various semantic labels against the factor dimension.

Particularly in market research, one needs to be able to map the decision-maker's peculiar perceived dimensions into more common (or at least one's own) perceived attribute dimensions. In Section IV, a radical solution to the probl of interpreting the attribute dimensions will be suggested.

The author has a strong appreciation of the merits of MDS; thus these remarks are meant merely to suggest some probable limits on economic feasability in many practical, complex situations, and to suggest directions for more efficient utilization of the decision-maker's time.

III. A Methodology for Decision Assumption Measurement

The methodology here contended to offer greater usefulness for most practical measurement problems of interest is an adaptation of Kelly's Role Repertory Test.[8] It is similar to an independent derivation now being introduced in

[8] See Kelly [11] and Mair [16]. The Role Repertory Test was originally developed to measure the structure of interpersonal social perceptions.

England for market research purposes, but is more general and utilizes a more powerful statistical modeling strategy.[9] It appears an extremely flexible method which, depending on the situation, has many of the desirable properties of either the semantic differential or multi-dimensional scaling.

The reader should refer to Appendix I for a detailed example of its use. This procedure first asks the decision-maker to match a given list of object role descriptions pertinent to a class of objects with appropriate objects from the decision-maker's own experience. A limited number of triads of these objects are selected. For each triad, the decision-maker is first asked which pair of the triad is most similar, and then in what important way they are similar with respect to which the third object is different. This first stage is identical, except for the superior way of choosing objects and for the additional step of eliciting of attribute names if available, with some forms of non-metric multi-dimensional scaling (MDS). However, instead of proceeding to ask for further comparisons among a large number of or among all possible triads of the group of objects, this first stage is ended at an earlier point and the second stage begun. At the second stage, Kelly had the subject to be measured, if, say, fifteen triad comparisons had been utilized in the first stage, assign each of the familiar objects to either the "similar" or "different" pole of each of the fifteen raw attributes implicitly defined by the first stage triad comparisons. The objects were then given a score of plus or minus one on each labeled implicit scale, and the attribute data thus obtained factor-analyzed to eliminate redundancies.

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[9] See articles by Lunn [15] and Morgan and Purnell [17] detailing uses and possibilities in market research.

The advantages of Kelly's procedure are, first, that the triad similarity comparison task which induces a high work-load is made easier by using individualized, self-selected familiar objects; second, that useful information in the form of implicit attribute labels is extracted; and third, that the first stage similarity comparison task is cut short after eliciting these labels and a more efficient method utilized to obtain data thereafter.

In the second stage, it is a relatively easy task for the decision-maker to position each object on each individual scale obtained in the first stage labeled with his own vocabulary. This permits, in the author's opinion, more accurate and feasible recording of the initial data in most practical cases than does MDS, primarily because we are putting the decision-maker's own subjective internalized semantic structure to work.[10]

The present investigator has altered Kelly's procedure at the beginning of the second stage by first allowing the subject to divide the newly labeled attribute scale into from two to nine equivalence intervals, depending on how fine a discrimination the subject wishes to make. The objects are then coded as falling into the appropriate equivalence interval before being factor analyzed. Objects are coded from 0 to 1000; for example, if two intervals are used, objects are coded either 250 or 750. Separate intervals for "not enough information" or "scale not appropriate" are also provided.

Factor analysis using this data for a single decision-maker thus eliminates most of the previously mentioned difficulties of the semantic differential, since the attributes are much more likely to be relevant, since a separate

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[10] This would not be true in those cases where the subject could not be induced to produce signs or labels indicative to him of the underlying attribute. However, such cases are rare, if they exist at all.

analysis is run for each decision-maker, and since the factor analysis may be based, if necessary, on purely ordinal data with only two equivalence intervals per scale.[11]

If the decision-maker can easily distribute, say twenty, objects on a relatively well divided scale, we obtain a great deal of metric data from him. Such a task may be nearly equivalent in value to getting him to make all possible (190) pairwise ordinal or even metric comparisons, but it is of much lower cost. When this is not possible, the decision-maker will have a tendency to divide the scale less finely, so that the methodology is self-corrective in the degree to which it forces metric quantification directly on the decision-maker's responses. Further, it can use as much ordinal or metric information as is available on each scale separately, rather than making one assumption for all attributes, as does MDS.

The separate factor analysis for each decision-maker of the resulting ratings of each object on each already elicited and decision-maker labeled (raw) attribute scale provides the ability to further refine the positioning of the objects on orthogonal factor attribute dimensions of higher explanatory power, and also to eliminate redundant raw attribute labels. The author has successfully used a varimax rotation of principal components as the method of factor analysis, but other reasonable factor analytic procedures could be substituted.[12]

The obvious next step here would be to use least-squares multiple

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[11] Purely ordinal data implies factor analysis of "dummy" variables; this presents some theoretical problems, but in practice factor scores based on such an analysis of real data typically have considerable metric content.

[12] See Dixon [5] and Harmon [7] for further information on factor analysis. Actually, "component analysis" is what is done, since "factor analysis", in the technical sense, would discard any variation present in only a single variable, a procedure which is not reasonable here since our only intent is to summarize the data.

regression of the objects' factor scores or their hypothesized transforms against separate ratings for the objects on some independently measured attribute. The regression coefficients would thus represent measures of assumptions. In particular, if the separate attribute is a decision variable, the resulting coefficients represent decision assumptions. If "dummy" decision variables are used, the procedure reduces to discriminant analysis.

Herzberg has recently gathered evidence through Monte Carlo simulation that regression based on principal components scores usually yields more dependable regression-based prediction equations than those estimated directly from a large number of raw variables, apparently because of the reduction of the classic problem of multi-collinearity between explanatory variables. The improvement is greatest for regressions using small numbers of observations.[13]

The author was not aware of Herzberg's results at the time of the study in which this methodology was developed; as a result the following very conservative procedure was constructed. It is recommended because of certain other advantages. Rather than rely on a single set of data for both factor analysis and regression coefficient estimation, two separate sets of data are used. From the first set, ratings on each of the raw attributes by the decision-maker are factor analyzed. The factor structure thus obtained is used to transform a second, new set of ratings of the same or similar objects on the originally elicited scales from the second data set into a set of quasi factor scores. That is, the factor scores are obtained which would have been obtained if the sample factor structure from the second data were the same as that obtained from the first data. The mathematics of this transformation are outlined in Appendix II. Derived in this way, the additional regression

[13] See Paul A. Herzberg [8]. See Johnston [10] for a straightforward discussion of the problem of multi-collinearity among the explanatory variables in estimating a least-squares regression model.

degrees of freedom created by the data reduction are *a priori* non-illusory. From a statistical view, since this method does not use all the correlation information in the two samples of data, it is not theoretically optimum; in particular, the order of the two data sets then could be reversed, and the second list of regression coefficients thus obtained could be combined with the first to produce a joint estimate. Also, it is not even clear, to this investigator, that merely basing the regression and factor analysis on a doubly-large set of observations would not be of equal benefit, when considered on purely statistical grounds.

However, in the particular study used as context for developing the methodology, the conservative, two-sample approach offered an overiding advantage. The first set of data could be taken on objects familiar to the individual decision-maker, but the second set, since the factor-structure had been already estimated, could be taken using a common set of objects for a large set of decision-makers; here familiarity with the objects was no longer as critical. Thus, the possibility of detailed comparison of assumptions among decision-makers is left open. This will often be desirable in practical situations.

Typical least-squares regression provedures relating the quasi factor scores or their transformations to the separately measured decision preference ratings under study would complete the measurement of decision-assumptions. However, when the available degrees of freedom are small, as they will generally be when using observations taken from real managers rather than graduate student subjects, problems will occur in obtaining reliable regression coefficients. Of course, the reliability of the measure can be tested against new data, but the author's experience suggests another way. Some reasonable confidence may

be placed in the measure, even if quite small samples of observations are used, if care is taken not to contaminate the analysis. The author has used twenty observations for factor analysis and another twenty for regression estimation for each decision-maker. The key in such small sample situations is a stringent algorithm for obtaining the regression equation with no utilization of subjective judgement, but rather utilizing objectively applicable a priori heuristics and statistical tests. In coarse outline, the algorithm suggested is, first, to allow factor scores of statistically-significant factors only; second, to enter these as variables one at a time into the regression equation in order of descending eigenvalues obtained in the original factor analysis; and third, of course, to use a test of statistical significance in the regression before entering the factor into the regression equation. The rare case of serious multicollinearity (since the factor scores are only "quasi" they will sometimes be correlated; this is often a sign of inadequacy of the original factor analysis) or the occasional lack of regression significance of any of the estimated factors may be handled by appropriate sub-algorithms.

Monte Carlo simulation analysis of the likelihood of various factor analysis obtained eigenvalues is suggested as most feasible for testing factor significance. Also, it may be advantageous to use a regression significance test based on the possibility of errors in the explanatory variables, rather than just in the dependent variable, although the author's experience did not suggest this difference is especially critical. The heuristic of using the most important factor first, as measured by its original eigenvalue, on the other hand, did appear to be critical in resolving problems of serious multi-collinearity or apparent lack of any factor's contributing to the re-

gression explanation. Also, the use of a pre-determined sequence of trials in entering explanatory variables allows a conservative regression significance test which eliminates as available degrees of freedom one degree for each variable previously tried, even if it was rejected. This eliminates the fairly well-known illusions of inflated significance typically experienced during step-wise regression.[14]

IV. A Method for Solving the Interpretability of Dimensions Problem

A detailed exploration of this topic is beyond the scope of the paper. In brief, however, problems of this sort can be approached through the parallel application of the measurement methodology to both the decision-maker to be measured and the outside observer who wants to interpret the subjective dimensions of the decision-maker. At the point of gathering a second set of data to be transformed into quasi factor scores, both observer and decision-maker use the same real-world referent objects. Thus, two different sets of quasi factor scores will be derived, based on the same referents. If the inter-relationships between the factors are analyzed by the method of canonical correlations, the resulting canonical correlation coefficients will provide the best linear mapping of the attributes used by the decision-maker into those used by the observer.[15]

V. Practical Feasibility and Conclusion

In the only case extensively studied by the author, the method was prac-

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[14] These issues are explored in detail in Wilcox [25].

[15] See Anderson [2] for an exposition of canonical correlation.

tically useful in measuring the assumptions of twenty-five participants in the stock-market regarding criteria for common stock suitability for particular investment objectives. Appendix I illustrates typical intermediate results. This usefulness was tested by using the measures to predict preference or suitability ratings from a third data set three months later. For totally new stocks, the average prediction power of the predictive equation based on the assumption measures for the new suitability ratings was $R^2 = 0.31$, with twenty observations. Many of the participants were mutual fund portfolio managers, security analysts, bank investment officers, and trust officers. The original Role Repertory Test took, in this case, an average of less than an hour. The ratings used for factor analysis and regression were done using mailed questionnaires, but reports indicate less than two hours, probably only an hour, was spent on average for each of these. In all, the average time spent by the decision-maker to get the measure was probably no more than four hours. The data collection and statistical analysis, indeed the entire procedure, is otherwise mostly susceptible to computerization and can be done at this scale very economically. Given the objectivity of the results, and the low level of skill required of the measurer to enter the data into the analysis, the methodology seems to perform very favorably as compared with competitors, although decisive comparisons obviously await further experimentation. Thus, its utility in the problem context of managerial planning and control of the decision process, and also in operations and market research, appears likely.

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Appendix I

The following exhibits represent typical instruments and intermediate results of the measurement methodology as applied in a study of the decision assumptions of stock market participants as to the attributes determining suitability of the stock for investment.

Stock Role List

1. The stock in which you first made a substantial profit
2. The stock in which you first took a substantial loss
3. Your present favorite stock
4. The stock you most dislike
5. A stock which has gone up significantly
6. A stock which has fallen significantly
7. A very popular stock
8. A stock which may become a good buy sometime in the next year
9. A stock a friend likes which you don't like
10. A stock you like which a friend doesn't like
11. A stock you know little about
12. A stock recommended to you by a knowledgeable person
13. A stock which you would recommend to others
14. A stock which might make a good short sale
15. A stock whose market action you feel you understand
16. A stock whose market action is hard to understand
17. A stock which you should have sold sooner
18. A stock which you should have bought sooner
19. A stock which you should have waited longer to sell
20. A stock which you should have waited longer to buy

A few of the individual investors had difficulty in thinking of twenty different stocks which seemed to fit the roles well, but none of the professional market participants had this difficulty, although some of them expressed frustration at not being able to use the same stock in several different places. A sample size of twenty stocks was chosen, somewhat arbitrarily, because it seemed like a reasonable compromise between having so many stocks that the interview would be too long and having so few that an insufficient variety of stocks would be included.

The following is a typical list of stocks elicited in this manner.

Subject No. 19, May 15, 1969, an individual investor:

1. Long Island Lighting
2. Tri-Continental Warrants
3. Shell Transport and Trading
4. Brooklyn Union Gas
5. American Greetings Corp.
6. Nortek
7. I.B.M.
8. Bow Valley
9. Pet Inc.
10. Avco
11. Avnet
12. Lehman Corp.
13. Federated Department Stores
14. Food Fair
15. Public Service of Colorado
16. Brunswick
17. Anglo-Lautro
18. Tri-Continental Common
19. Dexter Corp.
20. Hawaiian Airlines

List of Triads to be Sorted

(The role numbers identify the object)

1 - 4 - 15	11 - 8 - 15
2 - 6 - 19	12 - 2 - 1
3 - 4 - 14	13 - 18 - 17
4 - 2 - 11	14 - 12 - 13
5 - 7 - 8	15 - 6 - 16
6 - 19 - 10	16 - 4 - 11
7 - 9 - 10	17 - 20 - 18
8 - 3 - 5	18 - 2 - 9
9 - 10 - 5	19 - 3 - 11
10 - 17 - 4	20 - 12 - 3

The excerpts below, transcribed from tape-recordings, are typical responses to the sorting exercises.

Sample Interview Transcript Excerpt

Subject No. 37, May 20, 1969, a security analyst, sorting Falstaff Brewing, Needham Packing, and Digital Equipment:

> Subject: In a vague way you can organize these according to their business, again, Falstaff Brewing and Needham Packing both being in segments of the food industry, Digital Equipment involved in computer applications. The -- I would look next at management, and because Digital is one I'm not close to, I can't evaluate their management. Falstaff, again, is weak, here, and Needham is new and untested, operating -- it's a company that has a concept

and is trying to develop it. I don't think these fall together very neatly in any respect ....

Researcher: How would that affect your attitude toward a stock to know it was, let's say, computer versus food?

Subject: Again, to answer that I would try to quantify the growth rate, the growth potential, of their industries, and I know that the computer -- that the segment of the computer business that Digital is in -- has a very rapid growing potential, as opposed to the brewing industry, which is perhaps 12% to 15% a year, and the meat packing business, which could be in the 15% range also, which Needham's concepts proved realistic. So the answer is -- you would think that if, all things being equal, that Digital would have a greater potential, but on the other side if they don't -- if they're not a quality management and so on, there could be much greater risk as well.

The researcher took from the tape recorded interview notes of the following type, which correspond to the transcription excerpts given earlier.

Sample Post-Interview Processing Notes

Subject No. 37:

Falstaff
Needham ——————————————————— Digital Equipment

food
industry ——————————— computer
applications

Falstaff
———————————————————

weak
management ———————————————

Needham
———————————————

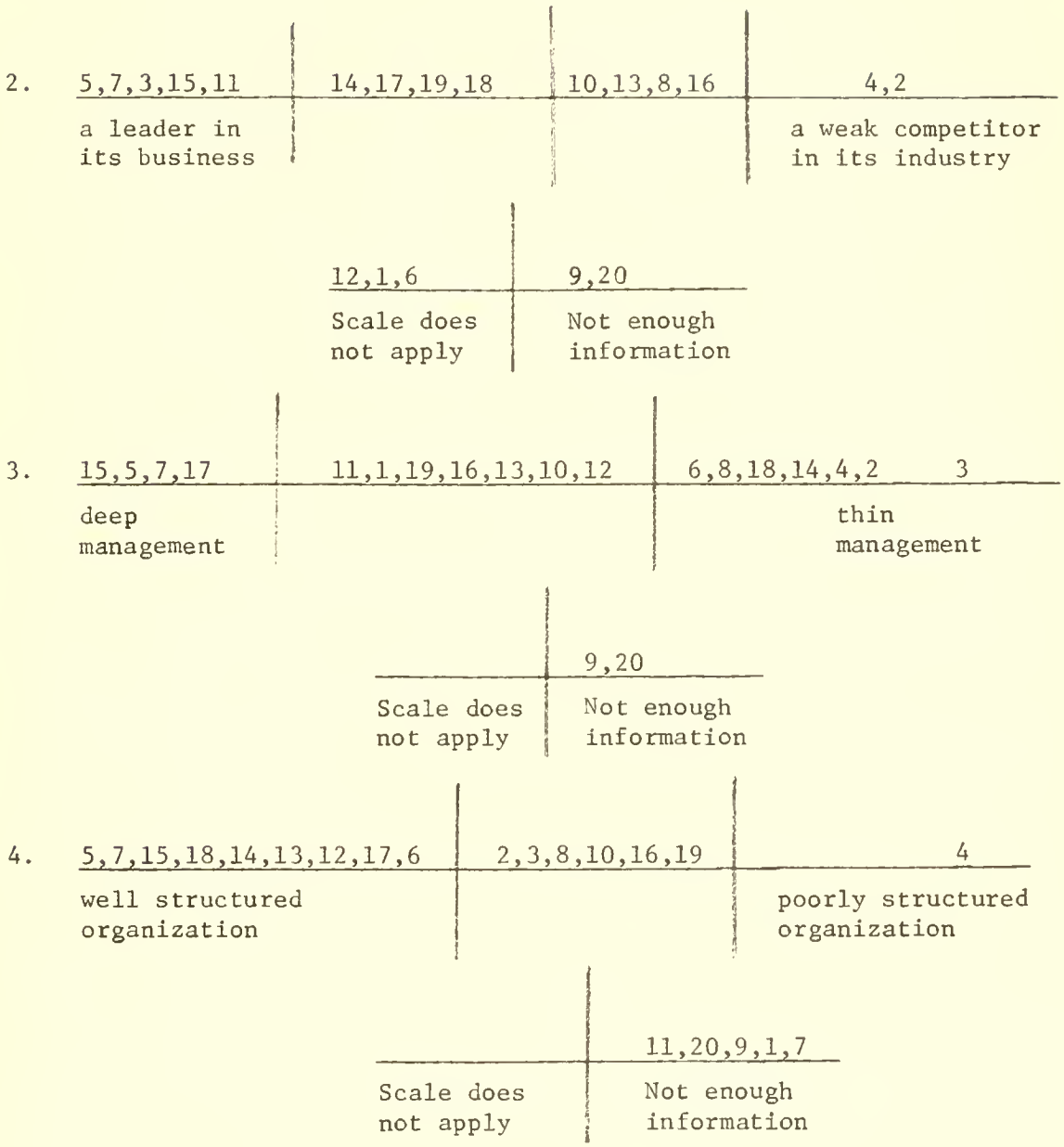
New, untested management ———
has a concept, trying
to develop it ———

Fallstaff
Needham ——————————————————— Digital Equipment

slower
——————————————— market growth
very rapid, high
growth potential

A questionnaire using these labels was filled out by the decision-maker as in the following example. The numbers stand for particular stocks previously elicited by matching the role list.

From the responses of subject number 37:



The responses to the foregoing questionnaire were factor analyzed. In order to avoid undue subjectivity, the factors were left unnamed. Factors which did not pass the significance test are marked not significant. The following example is typical: it refers to a single decision-maker's attribute dimensions. The pole at the left represents a greater positive value for the factor. "NV" refers to number of raw attributes, "E" to eigenvalue, and the numbers in the left-hand column are loadings.

Factor Structure for Subject 19

NV = 35:

Factor I, E = 6.6

.74	serves a declining market	---- serves rapidly growing market
.73	not so	---- sophisticated in the use of financial instruments
.87	growth record is poor	---- growth record is good
.82	has saturated its market	---- not so
.81	not solid	---- solid
.84	mgmt. not very capable	---- has very capable mgmt.
.88	moves into innovations far behind competition	---- moves into innovations far ahead of competitors

Factor II, E = 3.2

.93 stock's previous lower level was from non-recurrent event	---- stock's previous higher level was from non-recurrent event

Factor III, E = 4.6

.93 not a regulated utility	---- a regulated utility
.73 diversified product line	---- single product line
.85 not so	---- hard hit by high interest rates

Factor IV, E = 4.1

.78 stock has recently been performing (worse) than others in its group	---- stock has recently been performing (better) than others in its group
.93 stock has fallen rapidly in last six months	---- stock has risen rapidly in last six months
.87 in process of being re-evaluated downward	---- in process of being re-evaluated upward

Factor V, E = 2.4, not significant (N. Sig.)

.83 listed on major exchange	---- over the counter
.77 not so	---- price has remained at the same level for last 2 or 3 years

Factor VI, E = 3.1

.95 few factors affecting profit are outside company's control	---- many factors affecting company profit are outside company's control
.70 would be hard hit by inflation	---- a hedge against inflation

Factor VII, E = 3.8

.73 basic commodity product (demand not volatile)	---- product demand is volatile
.72 I know and have investigated myself	---- not so
.85 not so	---- involved in foreign operations
.78 stock price governed by real underlying forces	---- stock price governed by paper forces, incestuous rumors

Factor VIII, E = 1.9, N. Sig.

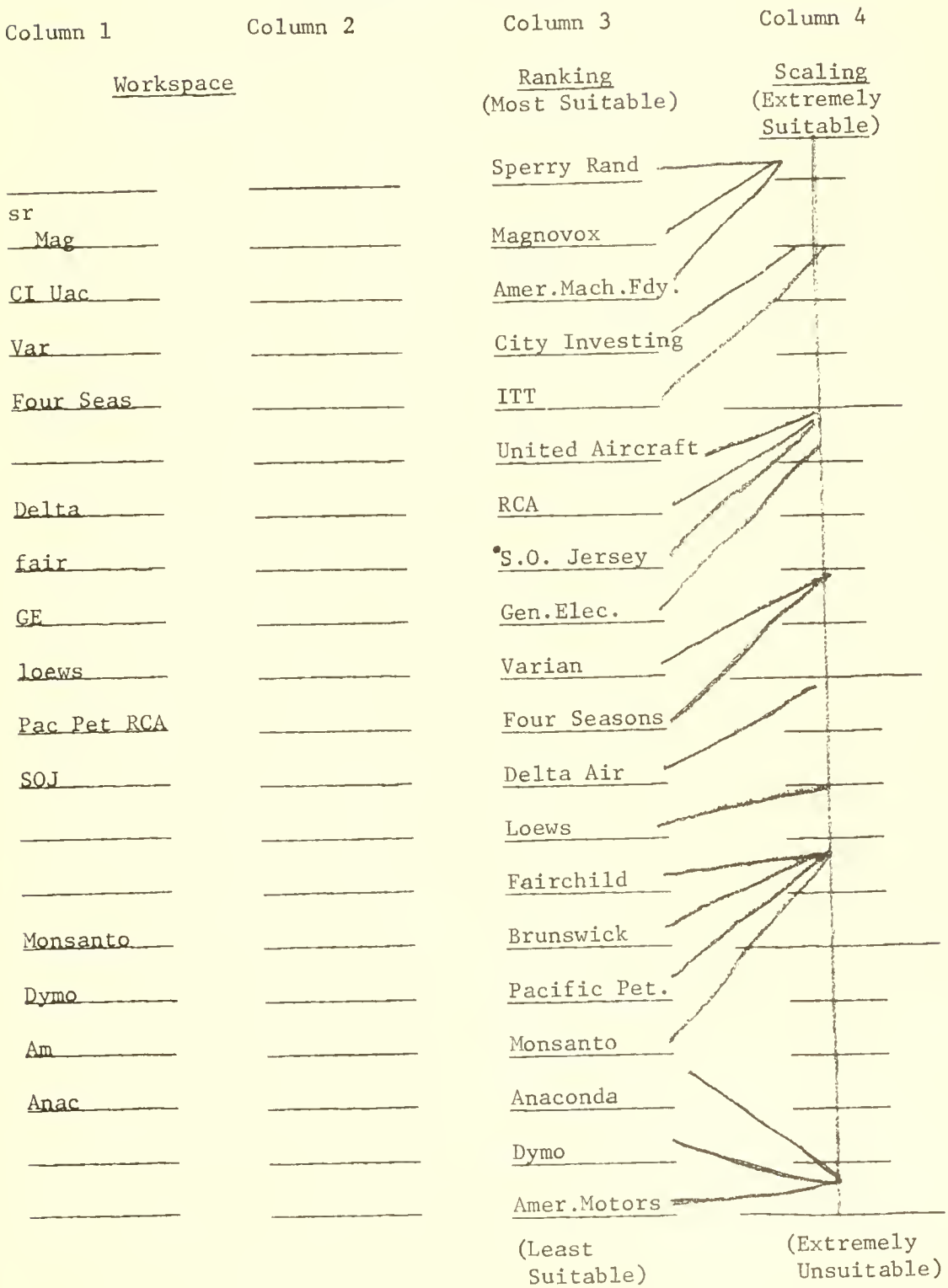
.84 stock very low compared to where it has been until recently	---- stock very high compared to where it has been until recently

Factor IX, E = 1.4, N. Sig.

.89 expresses concern for stock holders	---- not so

The following is a typical response to the preference rating task.

Response of Subject 30



The following is a somewhat better than average regression equation predicting preferences codes (Y) obtained for subject 19, whose factor structure was illustrated earlier. Over eighty percent of the variance in preference ratings was explained by factors one and four. This model held up well when tested on a new data set.

$$Y = 1690 - F_1(1.54) - F_4(0.78)$$

The particular set of regression coefficients illustrated above implies that at the time of testing subject 19 assumed a good management, growth, and market factor (Factor 1) implied suitability of the stock, and also, though with somewhat less fervor, he assumed recent favorable stock price performance was a favorable sign (Factor 4). Other attributes seemed to have no significant bearing on his decision as to suitability.

A complete description for each decision-maker of factor structures, assumption measures, and their tested predictive ability is available in Wilcox [25].

Appendix II

Use of Factor Measures to Condense the Data

The data from the first questionnaire were factor analysed in order to derive a transform which could then be applied to fresh data from succeeding questionnaires. Since the estimated factor structure was itself a small-sample observation, the resulting transform was imperfect compared to that which might be derivable from the population factor structure. However, the knowledge thus brought to bear, though imperfect, was distinctly useful; by reducing the number of explanatory variables, it made possible regression models upon which one could place some confidence. A maximum of three factor measure variables (quasi factor scores) was used in regression models based on twenty observations.

The derivation of the transforms may be expressed mathematically as follows. We have

$$\bar{\bar{Z}} = \bar{\bar{A}}\bar{\bar{F}},$$

where $\bar{\bar{A}}$ represents the n x m matrix of principal component loadings, with n variables and m factors, $\bar{\bar{Z}}$ the n x n' matrix of raw data attributes normalized to zero mean and unit variance, with n' observations, and $\bar{\bar{F}}$ the m x n' matrix of hypothesized principal component scores. In addition, we have obtained loadings for rotated factors, $\bar{\bar{B}}$, and corresponding rotated factor scores, $\bar{\bar{G}}$, where $\bar{\bar{B}} = \bar{\bar{A}}\bar{\bar{T}}$, $\bar{\bar{T}}$ an orthogonal transformation matrix. Thus, $\bar{\bar{Z}} = \bar{\bar{A}}\bar{\bar{F}}$ and $\bar{\bar{Z}} = \bar{\bar{B}}\bar{\bar{G}}$, where $\bar{\bar{B}} = \bar{\bar{A}}\bar{\bar{T}}$. None of the foregoing matrices need be square. We desire an estimate of $\bar{\bar{G}}$, given $\bar{\bar{Z}}$, $\bar{\bar{A}}$, $\bar{\bar{B}}$, and also given $\bar{\bar{\Lambda}}$, a diagonal matrix of the eigenvalues λ_k, (k = 1,m) of the matrix $\bar{\bar{A}}$. We know from the theory of matri

and factor analysis that $\bar{\bar{T}}' \bar{\bar{T}} = \bar{\bar{I}}$ and $\bar{\bar{A}}' \bar{\bar{A}} = \bar{\bar{\Lambda}}$. Thus we have:

$$\bar{\bar{B}}\bar{\bar{G}} = \bar{\bar{A}}\bar{\bar{F}}, \text{ since both equal } \bar{\bar{Z}}$$

$$\bar{\bar{A}}\bar{\bar{T}}\bar{\bar{G}} = \bar{\bar{A}}\bar{\bar{F}}$$

$$\bar{\bar{T}}\bar{\bar{G}} = \bar{\bar{F}}$$

$$\bar{\bar{T}}'\bar{\bar{T}}\bar{\bar{G}} = \bar{\bar{T}}'\bar{\bar{F}}$$

$$\bar{\bar{G}} = \bar{\bar{T}}'\bar{\bar{F}}, \text{ since } \bar{\bar{T}}'\bar{\bar{T}} = \bar{\bar{I}}$$

Now we want to find known expressions for $\bar{\bar{T}}'$ and $\bar{\bar{F}}$. From before we had:

$$\bar{\bar{Z}} = \bar{\bar{A}}\bar{\bar{F}}$$

and thus:

$$\bar{\bar{A}}'\bar{\bar{A}}\bar{\bar{F}} = \bar{\bar{A}}'\bar{\bar{Z}}, \text{ since we do not want to invert } \bar{\bar{A}}, \text{ which}$$

may not be square, and thus:

$$\bar{\bar{F}} = (\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{Z}}$$

Combining, we have:

$$\bar{\bar{G}} = \bar{\bar{T}}'(\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{Z}}$$

Finally, we can find $\bar{\bar{T}}'$ using:

$$\bar{\bar{B}} = \bar{\bar{A}}\bar{\bar{T}}$$

We have:

$$\bar{\bar{A}}'\bar{\bar{A}}\bar{\bar{T}} = \bar{\bar{A}}'\bar{\bar{B}}$$

$$\bar{\bar{T}} = (\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{B}}$$

$$\bar{\bar{T}}' = [(\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{B}}]'$$

Combining all terms, we have:

$$\bar{\bar{G}} = [(\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{B}}]' \ (\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{Z}}$$

$$\bar{\bar{G}} = \bar{\bar{B}}'\bar{\bar{A}}\,[(\bar{\bar{A}}'\bar{\bar{A}})^{-1}]' \quad (\bar{\bar{A}}'\bar{\bar{A}})^{-1}\bar{\bar{A}}'\bar{\bar{Z}}$$

$$\bar{\bar{G}} = \bar{\bar{B}}'\bar{\bar{A}}\,[\bar{\bar{A}}'\bar{\bar{A}}]^{-2}\bar{\bar{A}}'\bar{\bar{Z}}, \text{ since } (\bar{\bar{A}}'\bar{\bar{A}})^{-1} = [(\bar{\bar{A}}'\bar{\bar{A}})^{-1}]'$$

$$\bar{\bar{G}} = [\bar{\bar{B}}'\bar{\bar{A}}\bar{\bar{\Lambda}}^{-2}\bar{\bar{A}}']\,\bar{\bar{Z}}, \text{ since } \bar{\bar{A}}'\bar{\bar{A}} = \bar{\bar{\Lambda}}.$$

The final term in large brackets is a matrix which can be used to transform a new data set to quasi factor scores; it requires knowledge only of $\bar{\bar{B}}$, $\bar{\bar{A}}$, and $\bar{\bar{\Lambda}}$, and requires only simple matrix multiplication and transposition to calculate (since $\bar{\bar{\Lambda}}$ is diagonal, its inverse is just the matrix whose diagonal elements are the reciprocals of the elements of $\bar{\bar{\Lambda}}$).

This exact transform can often be closely approximated using only the first few columns of A, B and $\bar{\bar{\Lambda}}$. In practice m = n is typical, but we approximate the analysis using a smaller number of components.